Date Planned : __ / __ / __	Daily Tutorial Sheet – 15	Expected Duration : 90 Min
Actual Date of Attempt : __ / __ / __	Level – 3	Exact Duration :________

168. If B, C are square matrices of order n and if $A = B + C$, $BC = CB$, $C^2 = O$, then without using mathematical induction, show that for any positive integer $p, A^{p+1} = B^p\left[B + (p+1)C\right]$.

169. If $D = \text{diag}\left[d_1, d_2, ..., d_n\right]$, then prove that $f(D) = \text{diag}\left[f(d_1), f(d_2) .... f(d_n)\right]$, where $f(x)$ is a polynomial with scalar coefficient.

170. Find the possible square roots of the two –rowed unit matrix I.

171. If S is a real skew –symmetric matrix, then prove that $I - S$ is nonsingular and the matrix $A = (I + S)(I - S)^{-1}$ is orthogonal.

172. If f, g, and h are differentiable functions of x and $\Delta(x) = \begin{vmatrix} f & g & h \\ (xf)' & (xg)' & (xh)' \\ (x^2 f)'' & (x^2 g)'' & (x^2 h)'' \end{vmatrix}$, prove that

$$\Delta'(x) = \begin{vmatrix} f & g & h \\ f' & g' & h' \\ (x^3 f'')' & (x^3 g'')' & (x^3 h'')' \end{vmatrix}$$

173. Let α be a repeated root of a quadratic equation $f(x) = 0$ and $A(x), B(x), C(x)$ be polynomials of degrees 3, 4, and 5 respectively, then show that $\begin{vmatrix} A(x) & B(x) & C(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$ is divisible by $f(x)$, prime ($'$) denotes the derivatives.